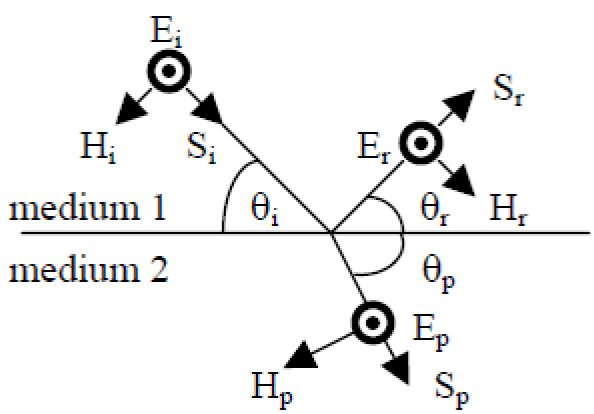
Brewster Angle, Pseudo-Brewster Angle, Polarization, Low Band Vertical Antennas and Elevation Patterns   N6MW 3/19/2016

While much has been written on the current topic, there remains a significant level of uninformed thinking in the ham world. The mission of this note is to clear up, where possible, the state of knowledge, though it perhaps is a bit pedantic.

First the Brewster angle technicalities. Prof. Brewster's angle concerns the reflection variation with angle of incidence of a plane wave off a plane surface between two media with different indices of refraction, i.e., purely dielectric media with different dielectric constants – for example, air and glass.

There are two different cases, perpendicular (perp) and parallel (para) polarization, from which any polarization can be formed. The perpendicular and parallel refer to the electric field, **E**, directions relative to the "plane of incidence" (POI) which is formed by the incident wave direction and the reflected wave direction. The refection is specular so the angle of incidence (we take it here as relative to the interface plane surface) is equal to the angle of the reflected wave. The magnetic field intensity, **H**, in a plane wave is orthogonal to **E** and both are orthogonal to the direction of propagation which is in the direction of the Poynting vector **S** = **E** X **H**.   If the polarization has **E** in the POI this is the parallel polarization case (where **H** is then perpendicular to the POI). If the polarization has **E** perpendicular to the POI this is the perpendicular polarization case (where **H** is then parallel to, i.e., in, the POI). The polarization is not changed by reflection. For the **"perp"** case Dzajic and Gacanovic (D&G hereafter) give the figure below. If **E** and **H** are switched (with **H** into the paper), that would be the para case. The θ here is the incident angle relative to the surface rather than relative to the normal.



D&G provide the full set of reflection amplitudes and phases for non-magnetic material that is both dielectric and conducting with material1 being air. The coefficient of reflection results are

$$\rho_{perp} = \frac{\sin\theta - \sqrt{(\varepsilon_r - j\chi) - \cos^2\theta}}{\sin\theta + \sqrt{(\varepsilon_r - j\chi) - \cos^2\theta}}$$

and

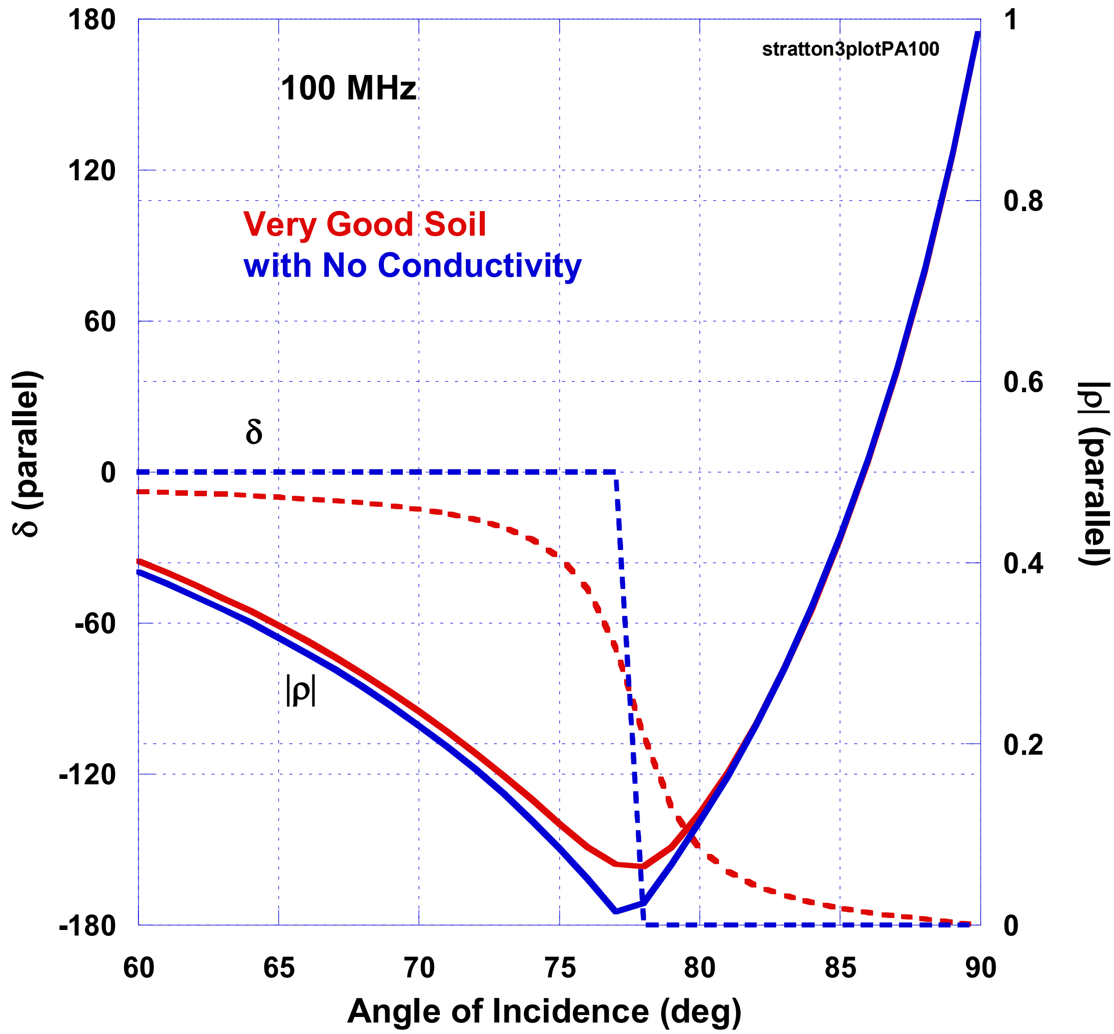
$$\rho_{para} = \frac{(\varepsilon_r - j\chi)\sin\theta - \sqrt{(\varepsilon_r - j\chi) - \cos^2\theta}}{(\varepsilon_r - j\chi)\sin\theta + \sqrt{(\varepsilon_r - j\chi) - \cos^2\theta}}$$

where $\chi=\sigma/\omega\varepsilon_0$, ε_0=8.85e-12, ε_r is the relative dielectric constant of medium2, σ is the medium2 conductivity in S/m and ω is the usual 2π times the frequency in Hertz. (A more general expression for any material properties, including magnetic, is provided by Stratton.) These complex coefficients provide, by definition, the relative electric field amplitudes and phases for the reflected wave (upper right in the figure) compared to the incoming wave (upper left in figure above).

For high frequencies, or low conductivity, χ becomes small and the **para** reflection coefficient goes to zero at $\sin\theta \sim 1/(\varepsilon_r)^{1/2}$. This angle, for no conductivity, for which ρ_{para} exactly vanishes is the famous Brewster's angle where there is no reflection off the surface.

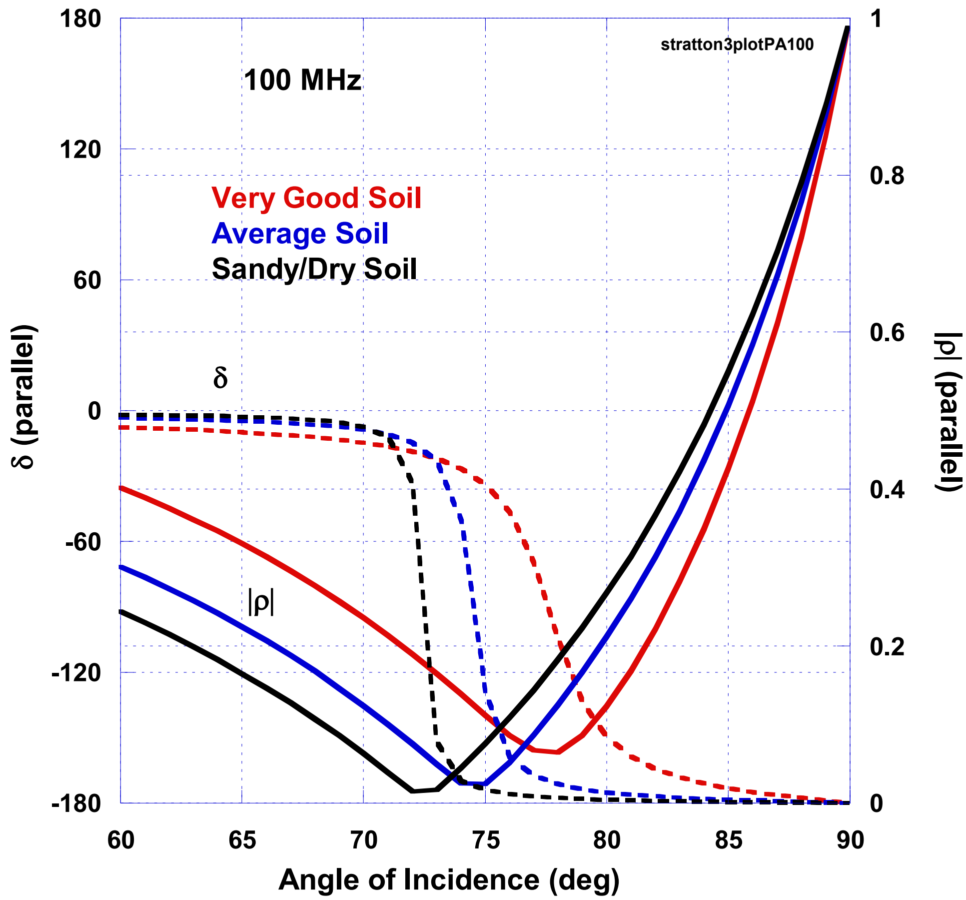
As an illustration, below is para reflection coefficient at 100 MHz for very good soil (σ=.03, ε=20) and the corresponding case with the same dielectric constant but no conductivity. In this plot and some of the following, the angle is relative to normal so 90 degrees is the grazing angle. The Brewster's angle is about 13 degrees above grazing for the no conductivity case (blue lines) and the phase change on reflection for angles nearer grazing is always 180 deg. The blue coefficient of reflection line would go to zero at the minimum if better resolution had been used in the plot.

Note that this means the reflection vanishes at the Brewster angle while it is finite below that angle (nearer grazing) but with the phase reversed. Above the Brewster angle the reflection is again finite but the phase is unchanged. Reflection is null only at the Brewster angle.

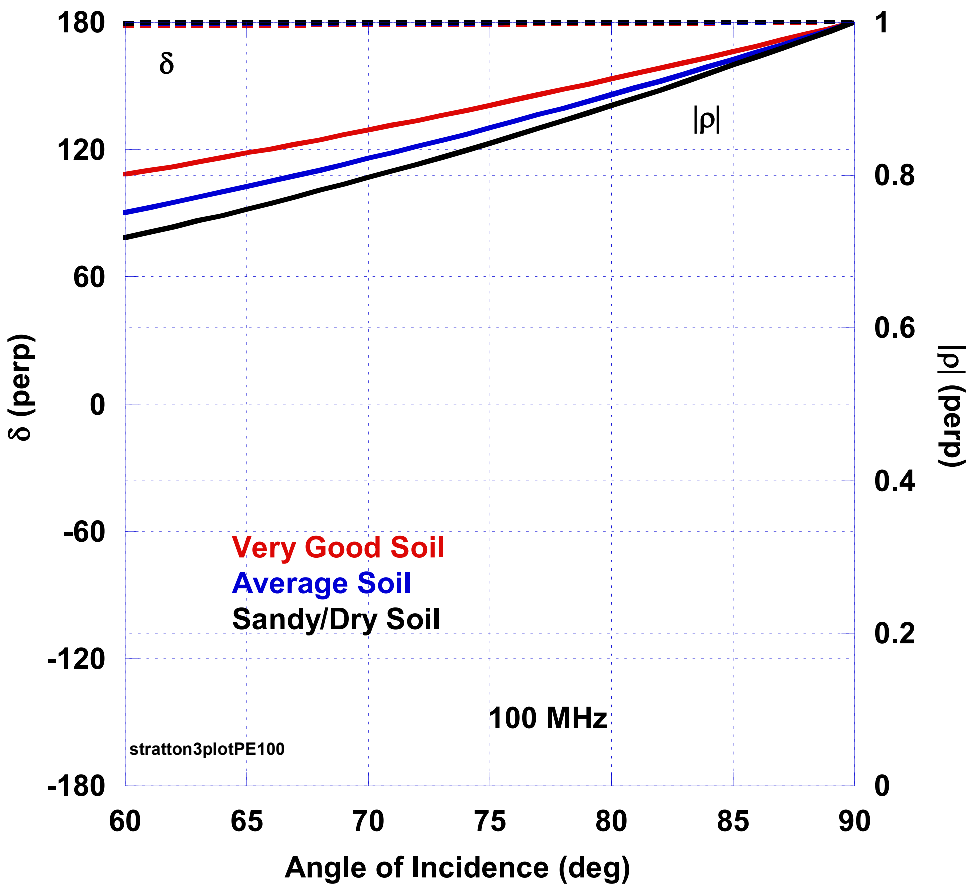


And The Pseudo-Brewster Angle – The plot above shows also the change that occurs when there is a modest amount of conductivity (so χ is not too big). The red lines deviate from the blue ones showing a non-zero minimum value of $|\rho|$ but at nearly the same angle for no conductivity. This angle for the minimum (for reasons lost to history?) is known as the Pseudo-Brewster angle. Otherwise, the curves are very similar but the phase change now varies smoothly with angle of incidence - not too surprising since the basic origins of the phenomena are the same. At near grazing angles, the two sets of curves are the same leading to the near cancellation of direct and reflected rays, to be discussed shortly, for both polarizations..

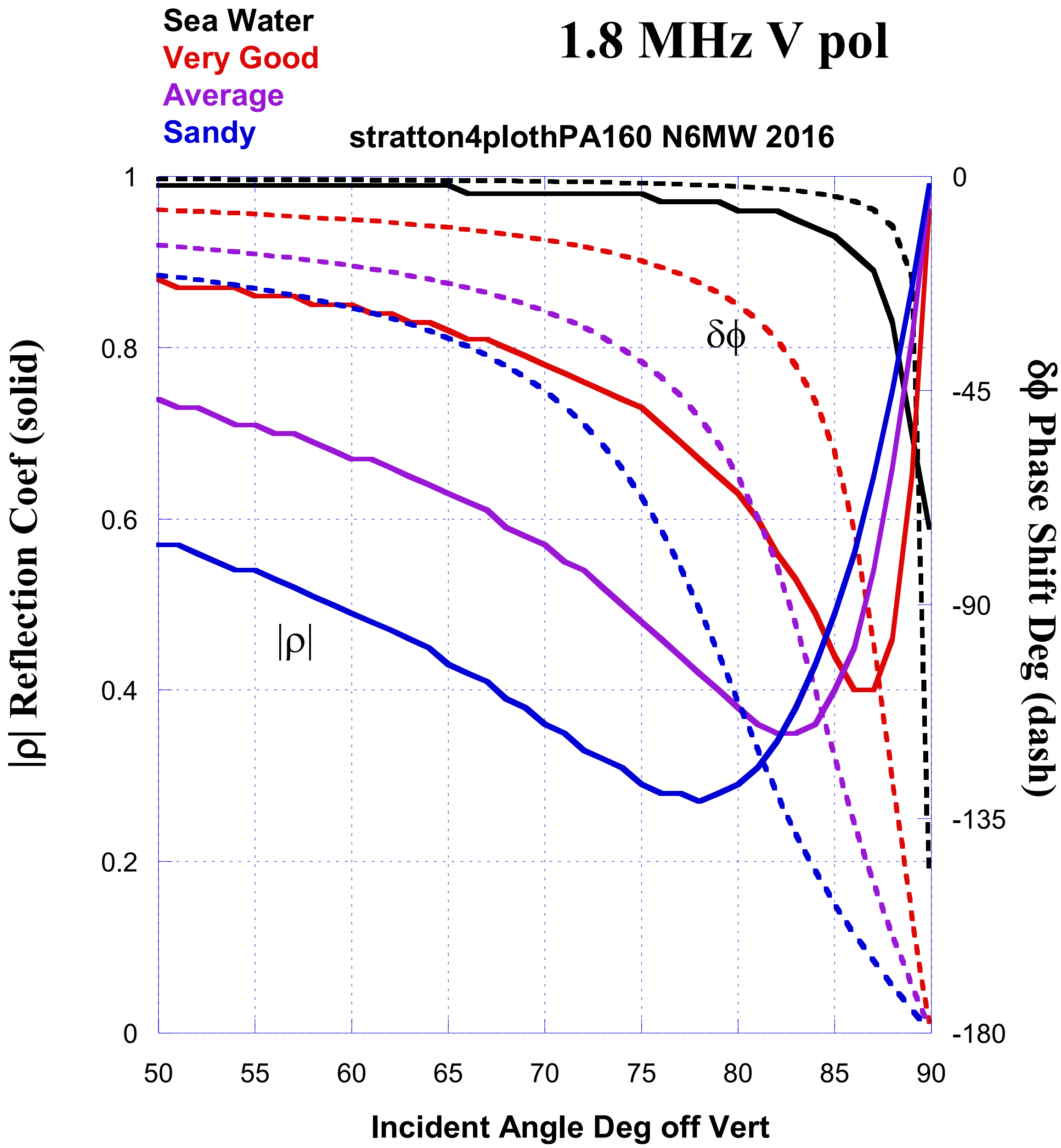
For a range of interesting soil conditions, at 100 MHz, the variation in para reflection is shown in the next figure with the curves moving closer to the non-conduction case with lower conductivity soil conditions.



For completeness, we next show the perpendicular polarization case for the reflection at 100 MHz for the same soil conditions. Here there is only a slow variation of ρperp and the phase is always reversed.



So How About at Lower Frequencies of Interest and Implications for Antennas – First we provide a set of curves for the **para** case ("V pol") at 1.8 MHz for the some set of soil types plus sea water. For the soils, lowering the frequency generally reduces the minimum and spreads out the phase change over more angle. The sea water case retains a large coefficient (near 1) of reflectance and small phase change up to angles very near grazing, as expected. In all cases the Pseudo-Brewster angle is evident as the minimum in the reflection coefficient curves. It appears (but not yet proved by me) that the phase change is 90 degrees at this Pseudo-Brewster angle.


Sea Water
Very Good
Average
Sandy
1.8 MHz V pol
stratton4plothPA160 N6MW 2016
|ρ| Reflection Coef (solid)
δϕ Phase Shift Deg (dash)
δϕ
|ρ|
1
0.8
0.6
0.4
0.2
0
0
-45
-90
-135
-180
50
55
60
65
70
75
80
85
90
Incident Angle Deg off Vert

So near the grazing angle the reflection is always at full amplitude and with a phase reversal for both para and perp. Therefore we expect both vertical antennas (para) and horizontal (perp) antennas over the earth will have a null in the pattern at the grazing angle. However, for the perp case (antenna horizontally polarized as in a horizontal dipole) the phase change (always reversed) is independent of angle so the pattern is less complicated and mostly dependent on the phase path difference with elevation angle as discussed next. In short, there is no Brewster-like effect for the perp case.

Okay, But How About the Elevation Pattern - At last we get to the real matter of interest. Notice that the last paragraph had nothing to do with any Brewster angle. Often discussions of Brewster for antennas stop at this point with a vague discussions of lower is better. So why do we care? The answer is about the elevation pattern of the antenna. Sure the pattern is null at grazing but how about higher angles – we like a nice low "takeoff angle" for DX Some may mutter a few words about reinforcement of the direct wave and carry on without further effort but we are not done yet. What we need to know is how do the reflected and direct waves add as vectors as a function of angle of incidence. Recall the angle of incidence equals the angle of reflected propagation. And except at grazing angle, there is a phase path difference between the direct and reflected wave and it depends on the angle. Happily this path difference is easy to compute, at least provided that the antenna is not right on the ground (more later on this). This path phase difference must be added to the reflection phase change to get the total.

If the emitter is at height h and the ground reflection occurs at a range d, the angle of incidence is $\theta=\tan^{-1}(h/d)$ and the phase path difference in radians between the direct and reflected wave (at long ranges) is

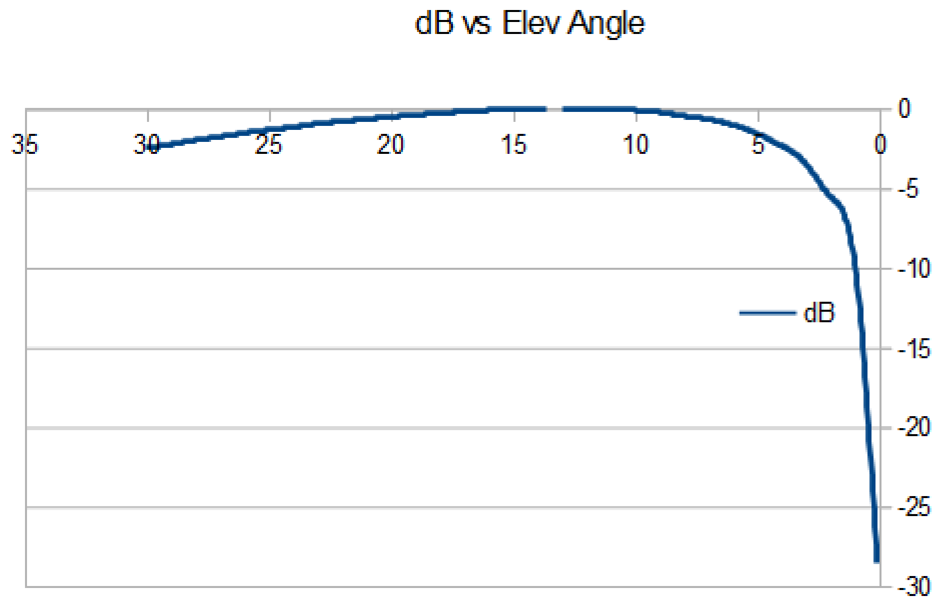
$$\Delta pp = \text{sqrt}(d^2+h^2)(1-\cos(2\theta))/\lambda$$

where λ is the wavelength. It is left to the student to verify. Finally the relative complex amplitude can be found by including the reflection phase change, as a function of angle as

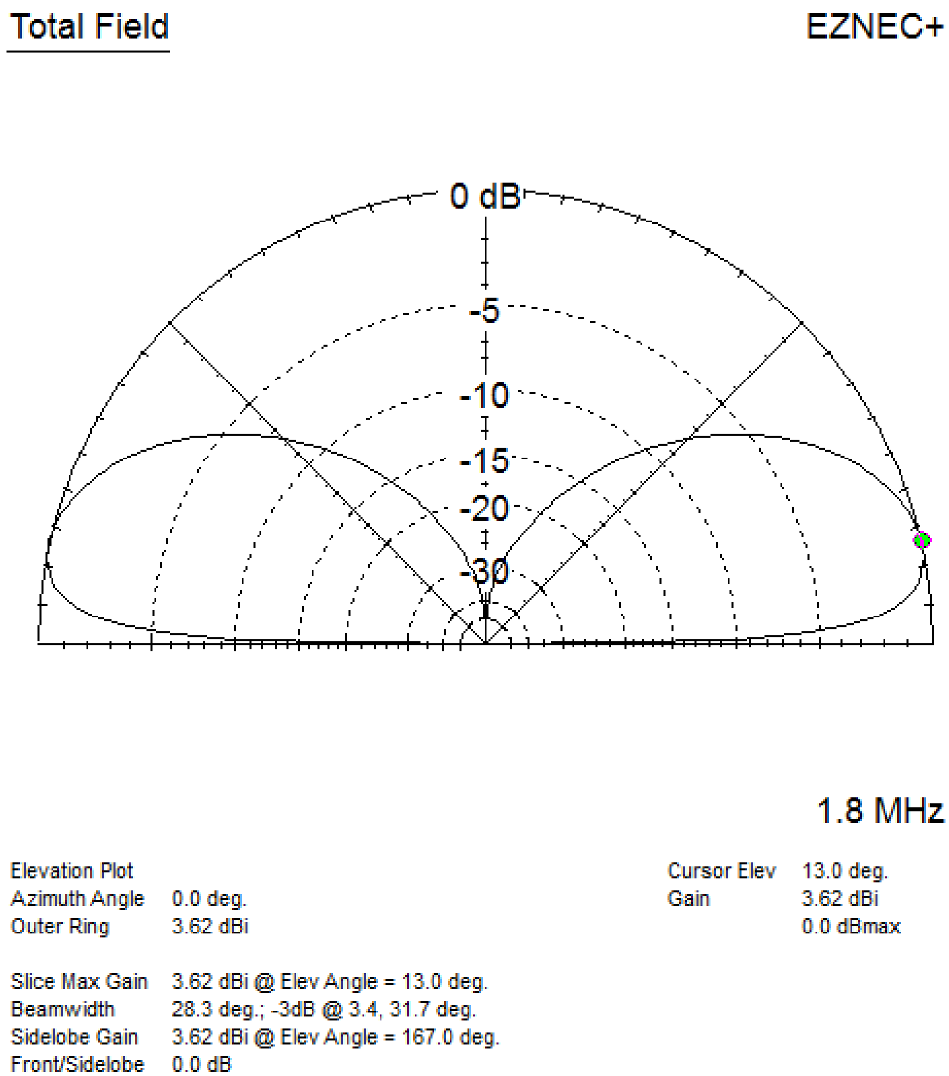
$$A = 1 + |\rho|\exp(j(\Delta pp+\delta))$$

A Side Comment: Some commenters describe this local reflection as the First ground reflection of the propagating wave where the perhaps Second and subsequent long range earth reflections (depending on the distance to the receiving antenna) are after refraction in the ionosphere. Calling the local ground reflection the First one is not a useful way of describing the propagation because the steady state long range signal at first "bounce" off the ionosphere is the distant sum of the direct and local reflected needed to get the full long range signal. The long range signal is not formed until well after the local reflection. However, note also that any subsequent reflection off the ground is subject to the same reflection coefficients described initially. Then the reflection phase changes play no role but the amplitude may be reduced if near the Brewster angle and over a poor conductor. Of course, the polarization of the signal is changed by ionospheric propagation so it's not so simple.

The relative power propagated to a given angle is then $|A|^2$. As an example consider a vertical dipole at $\lambda/4$ center height (h) at 1.8 MHz which gives the elevation pattern below. The roll off at higher angles is due to a multiplicative cosine factor from the free space dipole pattern. For this case the elevation peak is at about 13 degrees which corresponds to a reflection range of about one wavelength. The Pseudo-Brewster angle is about 3.5 degrees. For this case, including the path difference, the peak is about 3 dB above the pattern at the PB angle. The details depend on all the parameters discussed but generally the PB angle is roughly where the pattern detectably falls off at lower angles. Therefore the PB angle provides some limited measure of the resulting elevation pattern, given a proper interpretation.



This can be compared to the EZNEC result for the elevation pattern which is shown below for the vertical dipole example. The peak is at about 13 degrees and the fall-off on both sides is much like the plot above. This is not necessarily an endorsement of EZNEC to address this issue since it is said that EZNEC arrives at a pattern using the same considerations.



In the vertical dipole example used, as indicated before, the reflection point that corresponds to the elevation pattern peak (13 degrees) is about one wavelength from the antenna. This is well outside any normal radial field size and therefore the reflection is exclusively dependent on the character of the soil in a given direction at that range. (Strictly speaking, one should talk about a Fresnel zone rather that a reflection point but that is another discussion.) Generally we have no control of this reflection beyond selection of the antenna site so improving the elevation pattern beyond that is not really possible.

For comparison, a horizontal dipole at the same height has only a very broad peak in elevation pattern straight up and, of course, a null at grazing.

A Caution for Elevation Patterns for Ground Mounted or Low Antennas – As recently amplified on by Zarvel and noted from less formal comments by others on the Contesting Top Band and other forums, the use of ray optics for low antennas to estimate elevation patterns by employing Brewster related information is suspect at best. That is because the region from which the local reflections come is pretty close to the antenna and thus likely to be significantly distorted by near-field effects from the assumed freely propagating plane waves. As mentioned above, even for an antenna a quarter wave high, the example used before shows an elevation pattern peak arising from a reflection from a point just one wavelength from the antenna. At one λ range the fields are starting to approach their far field character in that case but for ground mounted vertical, which has maximum emissions very near the ground, ray optics become strained for pattern evaluation since the "reflection" points are now very close to the antenna., even if they are nearer the radial field. Furthermore, if the soil is not as good as in the example, the reflection point that leads to the resulting higher angle elevation peak then moves even closer to the antenna making the near/far field problem worse. Probably we should just take Brewster angle use as a rather broad guide for low vertical antenna evaluation (unless we receive actual data).

Even greater caution should probably be taken for the much discussed near-the-ocean-edge-vertical matters which then involves antennas over adjacent soil/high water table/water boundaries, the possible use of the water as a counterpoise (generally agreed a good idea) and the nominal local reflection point on the water in some directions.

References

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Julius A. Stratton, "Electromagnetic Theory," McGraw-Hill Book Company, New York, 1941.

Robert Zarvel, Elevation and Pseudo-Brewster Angle Formation of Ground-Mounted Vertical Antennas, QEX March/April 2016.